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The ABFST model and high energy scattering

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Abstract. We study the exact form of the ABFST integral equation treating it as an equation for the irreducible kernel, given the total $\pi\pi$ cross section. The usual identification of the irreducible kernel with the elastic cross section is seen to be highly unlikely. The result depends violently on the off-mass-shell behaviour of the $\pi\pi$ amplitude. In general the irreducible kernel is not even positive definite.

1. Introduction

Over the past few years a great deal of theoretical effort has gone into trying to compute the Regge trajectories and residues for elastic scattering on the basis of models for the $2 \rightarrow n$ production process. Historically the first model was due to Bertocchi *et al* (1962) and Amati *et al* (1962). The starting point for their model was the claim that the dominant Feynman diagrams for the production of n-pions were those with the maximal number of exchanged pions in the 't channel' as in figure 1. Thus at each 'vertex' down the chain only two pions come out. Using unitarity this leads to the equation shown schematically in figure 2.

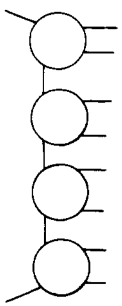

Figure 1. The ABFST production amplitude.

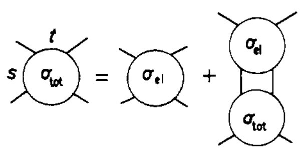

Figure 2. The ABFST integral equation.

From the work of Tow (1970) and Chew *et al* (1970), which we will discuss later, it turns out that the kernel of the integral equation shown in figure 2 is not big enough to give a constant total cross section as $s \rightarrow \infty$. The authors of the second paper first of all calculate the energy dependence of the forward elastic amplitude at high s with only low energy resonances in σ_{el}. This turns out to be $\mathrm{Im}\, A \sim s^{0 \cdot 2}$. The inclusion of the high energy tail in $\sigma_{el}(s)$ boosts this slightly but not far enough.

In this paper we wish to study this problem in reverse order. Thus we shall assume a specific form for the total $\pi\pi$ cross section and we shall solve the equation of figure 3 for the irreducible kernel I which, by definition, has no two pion states in the t channel. In order to make this definition unique we really need to know the off-mass-shell behaviour of the imaginary part of the forward elastic amplitude. Since we do not know this dependence we shall try various forms during our calculation.

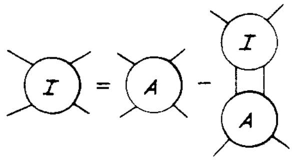

Figure 3. The equation defining the irreducible kernel. A is the imaginary part of the forward elastic amplitude.

In $g\phi^3$ perturbation theory which has been used extensively to serve as a model for high energy processes this irreducible kernel has one surprising feature. In the weak coupling limit, which is the only regime where a systematic asymptotic calculus exists (Eden *et al* 1966), the irreducible kernel does not contain any contribution from the leading trajectory. This is discussed in the Appendix. Extrapolating wildly we see that the high energy tail of the irreducible kernel may be very different from the tail of the elastic amplitude.

Originally we hoped that $I(s)$ might be small for large s so that a calculation ignoring the high energy tail would be meaningful. Then the input of $\sigma_{\mathrm{tot}}(s)$ for small s would give $I(s)$ for small s. Ignoring the high energy tail of $I(s)$ we could then solve for $\sigma_{\mathrm{tot}}(s)$ as $s \to \infty$ in a noncircular manner.

Unfortunately our irreducible kernel turns out to be large at moderately high s and so we need the input of $\sigma_{\mathrm{tot}}(s)$ at large s. The above sequence then becomes circular.

In this paper we shall calculate all quantities at $t = 0$.

2. The equation

This equation has been discussed so often that we merely quote the results. It takes the form:

$$I(s) = \mathrm{Im}\, A(s, m^2) + \int \mathrm{d}s'' F(s, s'') I(s'') \tag{2.1}$$

$$F(s, s'') = -\frac{1}{8\pi^4} \int_{4m_\pi^2}^{s_{\max}} \mathrm{d}s' \int_{u_{\min}}^{u_{\max}} \frac{\mathrm{d}u}{(u-m_\pi^2)^2} K(s, s', s'', u) \mathrm{Im}\, A(s', u) \tag{2.2}$$

$$K = \int \mathrm{d}^4 q' \delta(q'^2 - u) \delta((q-q')^2 - s') \delta((q'+p)^2 - s'') \tag{2.3}$$

with momenta labelled as in figure 4. Im $A(s, u)$ is the imaginary part of the forward amplitude with two of the particles off-mass-shell with $m^2 = u$. The term $I(s)$ is the two particle irreducible part of Im A. We shall assume that it has no off-mass-shell dependence. In order to determine any such dependence we would have to know Im A with

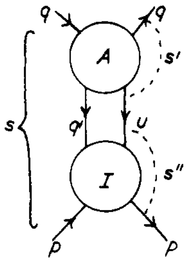

Figure 4. The momenta and invariants of the integral equation.

all four external particles off-shell. However, in (2.2) we shall examine various u dependences of Im $A(s', u)$.

In this paper we shall only consider the amplitudes corresponding to exchange of zero quantum numbers in the t channel. Thus we are looking at the pomeron.

Since we are interested in the low energy behaviour of $I(s)$ as well as its high energy behaviour we have made no t channel partial wave decomposition of any kind.

With our assumptions the equation for $I(s)$ becomes Volterra. Thus to compute $I(s)$ for $s < s_0$ we only need to know $A(s, u)$ for $s < s_0$. This means that when we reduce the integral to a gaussian integration so that the integral equation becomes a matrix equation the matrix is lower triangular. Its inversion is therefore numerically trivial. On the other hand if $I(s)$ is small compared to Im $A(s, m^2)$ for large s then we lose numerical accuracy rather quickly because of the large cancellations between the two terms on the righthand side of (2.2).

By looking at unitarity it is clear that $I(s)$ should be positive for all $s > 4m^2$. However, this is not guaranteed by the above equation and indeed we will obtain negative solutions. As input we have assumed an Im A with the following properties. At high energies it gives a constant total $\pi\pi$ cross section of 16 mb. At low energies we use the resonance parameters given in Chew *et al* (1970). The pomeron term is continued down to low energies to give a background term to the resonances in agreement with the Harari–Freund conjecture.

3. The solutions

In figure 5 the input Im $A(s, m^2)$ is shown as curve A. If no further u dependence is assumed for Im A then the irreducible kernel is shown in curve D. If Im $A(s, u)$ is set equal to zero for $|u| > 1$ GeV^2 and unaltered otherwise we get curve B. If we try and mock up the calculation of Chew *et al* (1970) and use this cut-off only for $s > 3$ GeV^2 then we get curve C. Thus we see that $I(s)$ is very near Im $A(s, m^2)$ for $s < 3$ GeV^2. In particular we see that the high mass resonances correspond to σ_{tot} rather than σ_{el}. Unfortunately our integration routines cannot cope with the resonances accurately for $s > 20$ GeV^2. However, they are contributing rather little to the integral for $s > 20$ GeV^2. In figure 6 we show the results of computing $I(s)$ at very much higher s with Im $A(s, m^2)$ given solely by the pomeron term. The line A corresponds to Im $A(s, m^2)$. The curve E corresponds to no u cut-off in Im $A(s, u)$ while B, C and D correspond to setting Im $A(s, u)$ zero for $u > 1$, 10, 100 GeV^2 respectively. We see in a rather startling manner how the u dependence is crucial in solving for $I(s)$ at large s. Indeed, the whole dynamics seems to depend violently on the off-mass shell behaviour. As a check on our arithmetic we

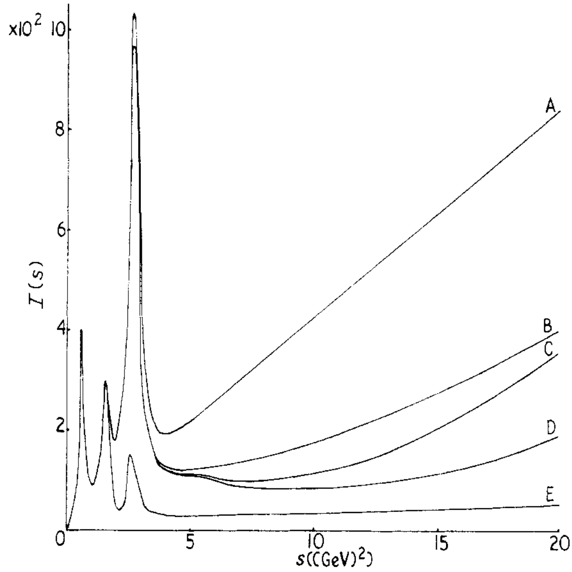

Figure 5. The input amplitude (curve A) and the irreducible kernel for various u cut-offs.

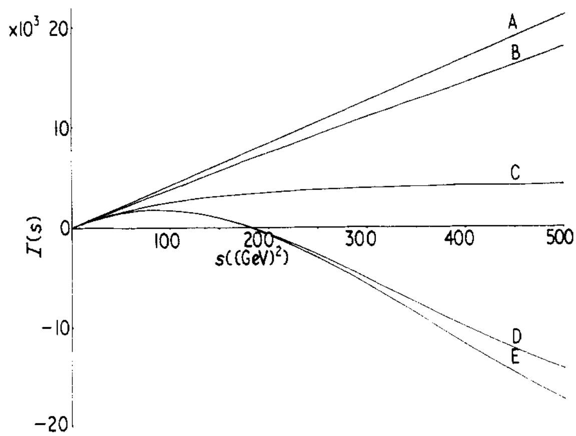

Figure 6. The irreducible kernel at larger s.

have solved for Im $A(s, m^2)$ with these irreducible kernels as input. They reproduce the original input to about 1 % accuracy. As a further check that our program would have found an irreducible kernel which tended to zero for large s we ran the program forwards and backwards with an input I consisting of a single resonance. The resonance residue was chosen to give a high lying trajectory when we solved for Im A. Putting the output Im A back into our equation and solving for I we obtained our input to reasonable accuracy.

In figure 7 we show the results of solving the equation with $I(s)$ given by $\sigma_{\text{el}}(s)$. Curve A corresponds to the irreducible kernel $I(s)$. If we put a 1 GeV^2 cut-off on u for $s'' > 3$ GeV^2 then B is the output Im A. This gives a leading output trajectory $\alpha(0) \sim 0{\cdot}4$. The curve C corresponds to using the u cut-off for all s''. Curves D and E correspond to multiplying the input resonances by 3·0 and 4·2 respectively while keeping the cut-off

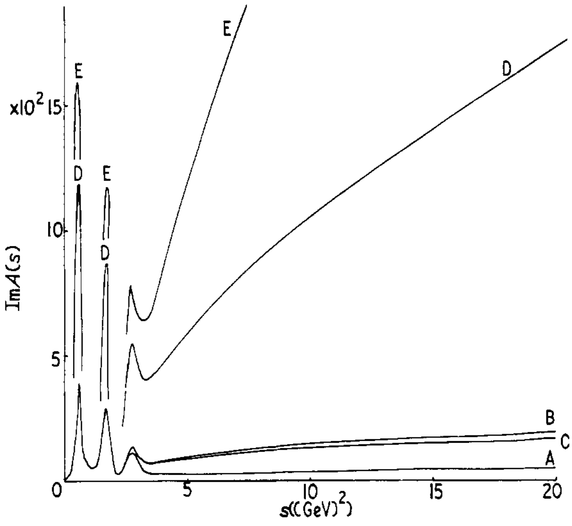

Figure 7. The output amplitude for various input irreducible amplitudes.

tail for $s'' > 3$ GeV^2. They correspond to leading trajectories at $\alpha(0) \sim 0{\cdot}7$ and $\alpha(0) \sim 1{\cdot}0$ respectively. Notice that the total cross section corresponding to the $\alpha(0) \sim 1{\cdot}0$ solution is about 80 mb. This agrees reasonably well with the estimate implied in Abarbanel *et al* (1970).

4. Conclusion

We thus see that the irreducible kernel as we have defined it is not a simple object. In particular its high energy behaviour is very dependent on the off-mass-shell behaviour of the $\pi\pi$ amplitude. It may be that if we considered a kernel irreducible with respect to K exchange as well as π exchange it would be well behaved. However, this means assuming something like SU(3) to determine the crossing matrices. Since K production in experiments is so low compared with the SU(3) predictions we cannot trust these numbers.

We have not examined the off-mass-shell dependence in any detail although it is clear from unitarity that this must exist in such a way that the irreducible kernel comes out positive.

Acknowledgments

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Appendix

First we define a different irreducible function I' by means of the schematic equation of figure 8. Then in the jargon of this subject I' has no d lines of length 1 (Eden *et al* 1966).

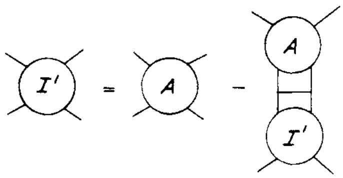

Figure 8. The equation defining $I'(s)$.

Thus we see that although

$$A \sim s^{-1+g^2K(t)+O(g^4)}$$

$$I' \sim s^{-2+g^2K'(t)+O(g^4)}.$$

Moreover, any graph in I is in I' and we do not expect cancellations in asymptotic behaviour. Thus we see that I does not contain the leading trajectory which occurs in A.

Very crudely one may say that if the t channel iteration of I is supposed to lead to the pomeron there is no reason why I should contain it.

References

Abarbanel H D I, Chew G F, Goldberger M L and Saunders L M 1970 *Phys. Rev. Lett.* **25** 1735–7
Amati D, Fubini S and Stanghellini A 1962 *Nuovo Cim.* **26** 896–954
Bertocchi L, Fubini S and Tonin M 1962 *Nuovo Cim.* **25** 626–54
Chew G F, Rogers T and Snider D R 1970 *Phys. Rev.* D **2** 765–80
Eden R J, Landshoff P V, Olive D I and Polkinghorne J C 1966 *The Analytic S-Matrix* (London: Cambridge University Press)
Tow D 1970 *Phys. Rev.* D **2** 154–63